Model Answer

AU - 5084

B.Tech. (V sem) Exam - 2014

Civil Engg. Branch (New course)

STRUCTURAL ANALYSIS - II

Q (1)

(i) — a

(ii) — a

(iii) — d

(iv) — a

(v) — c

(vi) — a

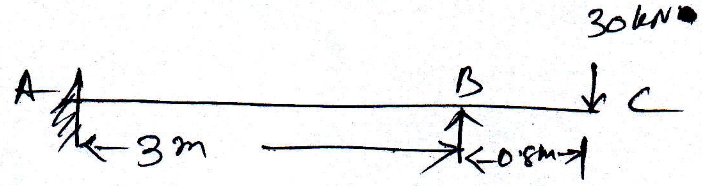
(vii) — a

(viii) — d

(ix) — d

(x) — c

Q2

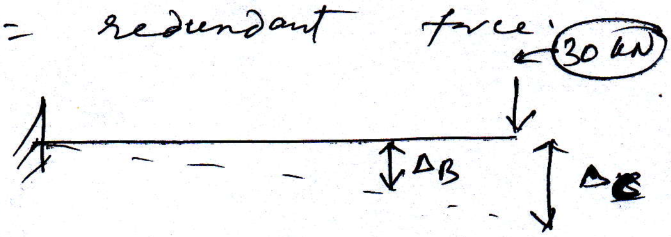


$EI = \text{const.}$

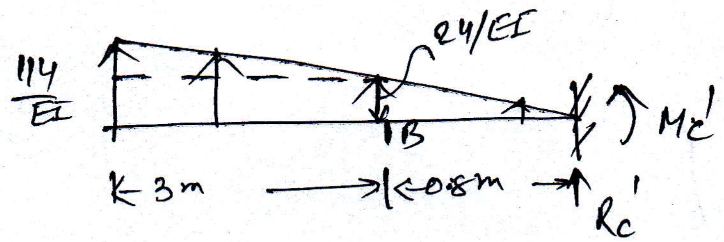
Consistent deformation method.

Let R_B = redundant force.

$R_B = 0$

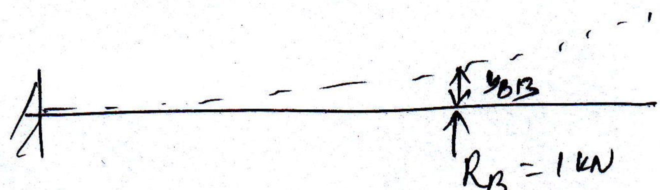


$$\Delta_B = \frac{WL^3}{3EI}$$



$$M_B' = \frac{1}{EI} \times (24) \times 3 \times \frac{3}{2} + \frac{90}{EI} \times \frac{1}{2} \times 3 \times \frac{2 \times 3}{3}$$

$$= 378/EI = \Delta_B$$

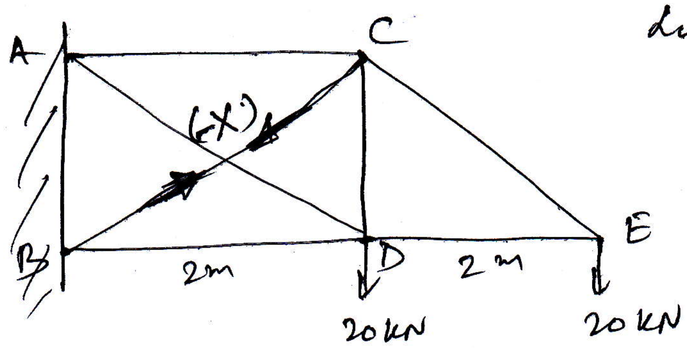


$$y_{BB} = \frac{3^3}{3EI} = \frac{9}{EI}$$

$$R_B = \frac{378}{9} = 42\,kN.$$

$R_A = -12\,kN$, $M_A = 12\,kNm$

Q3



Let X be the compressive force

$A = 1000\ mm^2$ for each member

$E = 200\ kN/mm^2$

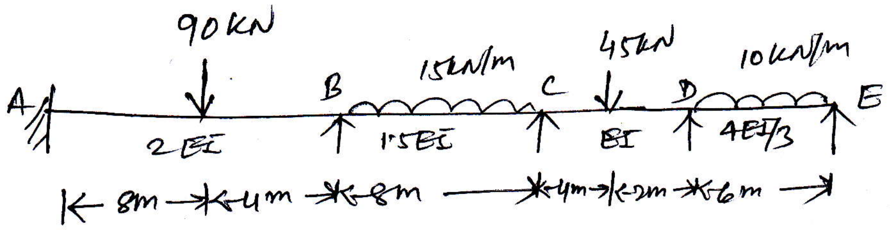
Let force in member B is the redundant force as X (comp) as true.

Member	F	$\frac{\partial F}{\partial X}$	$\frac{L}{AE}$	Forces in members (kN)
AC	$-20 - \frac{X}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$2/AE$	-40
CE	$-20\sqrt{2}$	0	$2\sqrt{2}/AE$	$-20\sqrt{2}$
ED	20	0	$2/AE$	0
DB	$60 - \frac{X}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$2/AE$	40
DC	$20 - \frac{X}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$2/AE$	0
AD	$-40\sqrt{2} + X$	1	$2\sqrt{2}/AE$	$-20\sqrt{2}$
BC	X	1	$2\sqrt{2}/AE$	$20\sqrt{2}$

$$\sum F \frac{\partial F}{\partial X} \frac{L}{AE} = 0$$

$$\Rightarrow X = +20\sqrt{2}\ kN \quad \text{(comp)}$$

Q4



FEM,

$M^F_{AB} = -80$ kNm, $M^F_{BA} = +160$ kNm

$M^F_{BC} = -80$ kNm, $M^F_{CB} = +80$ kNm

$M^F_{CD} = -20$ kNm, $M^F_{DC} = 40$ kNm

$M^F_{DE} = -30$ kNm, $M^F_{ED} = 30$ kNm

Slope deflection eq'ns.

$M_{AB} = -80 + \frac{4EI}{12}(2\theta_A + \theta_B)$, $\theta_A = 0$

$M_{BA} = 160 + \frac{4EI}{12}(2\theta_B)$

$M_{BC} = -80 + \frac{3EI}{8}(2\theta_B + \theta_C)$

$M_{CB} = 80 + \frac{3EI}{8}(2\theta_C + \theta_B)$

$M_{CD} = -20 + \frac{2EI}{6}(2\theta_C + \theta_D)$

$M_{DC} = 40 + \frac{2EI}{6}(2\theta_D + \theta_C)$

$M_{DE} = -30 + \frac{8EI}{18}(2\theta_D + \theta_E)$

$M_{ED} = 30 + \frac{8EI}{18}(2\theta_E + \theta_D)$

Also,

$M_{ED} = 0 \Rightarrow 2\theta_E + \theta_D = -\frac{67.5}{EI}$ ①

$\Sigma M_B = 0 \Rightarrow M_{BA} + M_{BC} = 0$

$\Rightarrow 1.416\,\theta_B + 0.333\,\theta_A + 0.375\,\theta_C = -\frac{80}{EI}$ ②

$\Sigma M_C = 0 \Rightarrow 1.4166\,\theta_C + 0.375\,\theta_B + 0.333\,\theta_D = -\frac{60}{EI}$ ③

$\Sigma M_D = 0 \Rightarrow 1.55\,\theta_D + 0.333\,\theta_C + 0.444\,\theta_E = -\frac{10}{EI}$ ④

Solving ①, ②, ③ & ④

$\theta_B = -\frac{48.10}{EI}$, $\theta_C = -\frac{31.68}{EI}$, $\theta_D = \frac{8.76}{EI}$, $\theta_E = -\frac{38.13}{EI}$

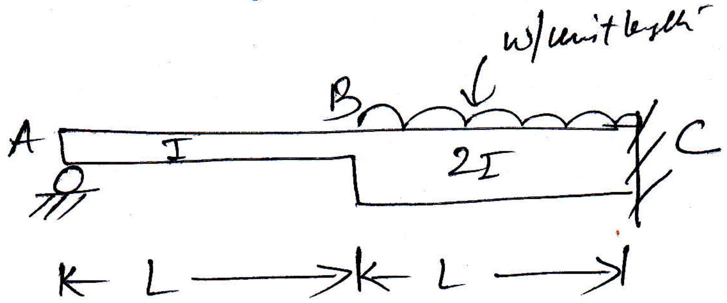
Hence

$M_{AB} = -96.1$, $M_{BA} = 127.7$, $M_{BC} = -127.8$ kNm

$M_{CB} = 38.1$, $M_{CD} = -38.1$, $M_{DE} = -35.2$, $M_{DC} = 35.3$ kNm,

$M_{ED} = 0$

Q5



$\theta_C = 0, \quad \theta_A, \theta_B, \quad \Delta_{AB} = \Delta_{BC} = \Delta$

$M^F_{AB} = 0 = M^F_{BA}, \quad M^F_{BC} = -\frac{wl^2}{12}$ & $M^F_{CB} = +\frac{wl^2}{12}$

$M_{AB} = 0 + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$

$M_{BA} = \frac{2EI}{L}\left(\theta_A + 2\theta_B - \frac{3\Delta}{L}\right)$

$M_{BC} = -\frac{wl^2}{12} + \left(2\theta_B + \frac{3\Delta}{L}\right) * \frac{4EI}{L}$

$M_{CB} = +\frac{wl^2}{12} + \left(\theta_B + \frac{3\Delta}{L}\right) * \frac{4EI}{L}$

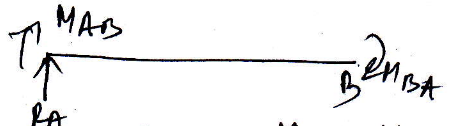
$M_{AB} = 0 \Rightarrow 2\theta_A + \theta_B - \frac{3\Delta}{L} = 0$ (1)

$\Sigma M_B = 0 \Rightarrow \frac{2EI}{L}\left(\theta_A + 2\theta_B - \frac{3\Delta}{L}\right) + \left(-\frac{wl^2}{12}\right) + \frac{4EI}{L}\left(2\theta_B + \frac{3\Delta}{L}\right) = 0$

$\Rightarrow \theta_A + 2\theta_B - \frac{3\Delta}{L} + 4\theta_B + \frac{6\Delta}{L} = \frac{wl^2}{12} \times \frac{L}{2EI}$

$\Rightarrow \theta_A + 6\theta_B + \frac{3\Delta}{L} = \frac{wL^3}{24EI}$ (2) $= \cancel{\frac{wl^2}{24EI}}$

Also, $\Sigma F_V = 0 \Rightarrow R_A + R_C = wL$



$R_A = -\frac{M_{AB} + M_{BA}}{L}$

$R_C \times L = M_{BC} + M_{CB} + \frac{wL^2}{2}$

$R_C = \frac{M_{CB} + M_{BC} + \frac{wL^2}{2}}{L}$

$$\Rightarrow -\frac{M_{AB}+M_{BA}}{L} + \frac{M_{CB}+M_{BC}+\frac{wL^2}{2}}{L} = wL$$

$$\Rightarrow -M_{AB} - M_{BA} + M_{CB} + M_{BC} = wL^2 - \frac{wL^2}{2}$$

$$\Rightarrow -\frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right) \quad \cancel{= \frac{wL^2}{2}}$$

$$-\frac{2EI}{L}\left(\theta_A + 2\theta_B - \frac{3\Delta}{L}\right) + \frac{wL^2}{12} + \frac{4EI}{L}\left(\theta_B + \frac{3\Delta}{L}\right)$$

$$-\frac{wL^2}{12} + \frac{4EI}{L}\left(2\theta_B + \frac{3\Delta}{L}\right) = \frac{wL^2}{2}$$

$$\Rightarrow \cancel{-4\theta_A - 2\theta_B}$$

$$\Rightarrow -2\theta_A - \theta_B + \frac{3\Delta}{L} - \theta_A - 2\theta_B + \frac{3\Delta}{L} + 2\theta_B + \frac{6\Delta}{L} + 4\theta_B + \frac{6\Delta}{L} = \frac{wL^3}{4EI}$$

$$\Rightarrow -3\theta_A + 3\theta_B + \frac{18\Delta}{L} = +\frac{wL^3}{4EI}$$

$$\Rightarrow -\theta_A + \theta_B + \frac{6\Delta}{L} = \frac{wL^3}{12EI} \quad - (3)$$

Solving (1), (2) & (3)

$$\theta_B = -\frac{wL^3}{96EI}, \quad \theta_A = \frac{11}{288}\frac{wL^3}{EI}$$

$$\Delta = \frac{19}{864}\frac{wL^4}{EI}$$

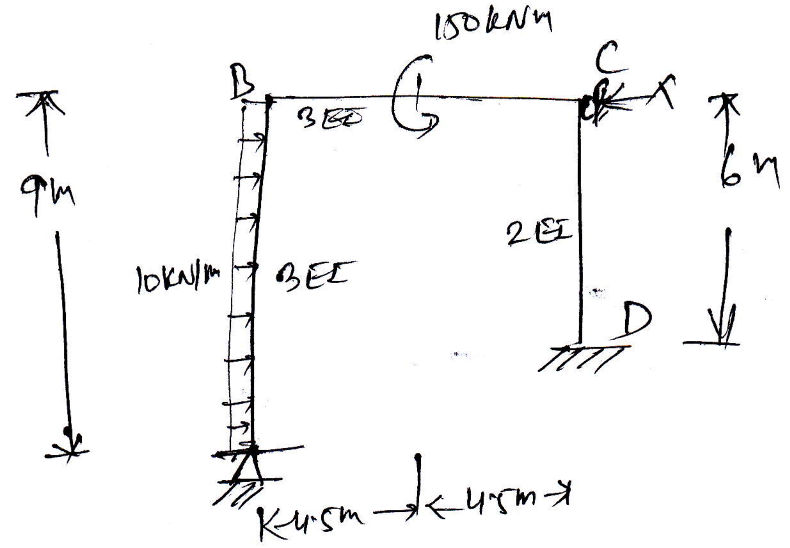
Hence, $M_{AB} = 0$

$M_{BA} = -0.0972\, wL^2$

$M_{BC} = +0.0972\, wL^2$

$M_{CB} = 0.3056\, wL^2$

Q6



$$M^F_{AB} = -\frac{10 \times 9^2}{12} = -67.5$$

$$M^F_{BA} = +67.5$$

$$M^F_{BC} = -\frac{M}{4} = -37.5$$

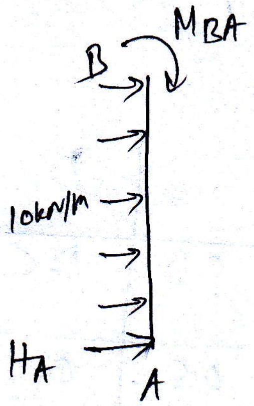
$$M^F_{CB} = -\frac{M}{4} = -37.5$$

kNm

$$M^F_{CD} = M^F_{DC} = 0$$

Non-sway moment distn

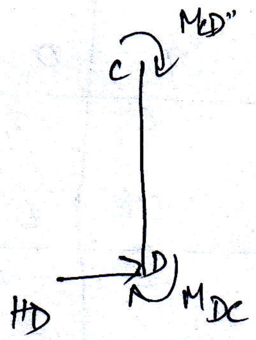
Joint	A	B		C		D
Mem	AB	BA	BC	CB	CD	DC
RS	I/3	I/4	I/3	I/3	I/3	I/3
DF	1	0.43	0.57	0.5	0.5	0
FEM	-67.5	+67.5	-37.5	-37.5	0	0
Bal	+67.5	-12.9	-17.1	+18.75	+18.75	0
COF	0.0	+33.75	9.4	-8.55	0	9.4
Bal	0	-18.55	-24.60	+4.27	4.27	0
			2.14	-12.3	0	2.14
COF	0.08	0		+6.15	+6.15	0
Bal	0	-0.92	-1.22			
COF	0	0	+3.08	-0.61	0	+3.08
			1.76	+0.31	+0.31	0
Bal	0	+1.32	0.15	-0.88	0	+0.16
COF	0	0	-0.086	+0.144	+0.44	0
Bal	0	-0.06				
Final	0	~~70.14~~ +67.5	~~70.14~~ -67.5 ~~67.5~~	~~[illegible]~~ -29.92	+29.92	14.78



$\Sigma M_B = 0$

$-H_A \times 9 - 10 \times 9 \times 9 \times \frac{1}{2} + M_{BA} = 0$

$H_A = \dfrac{67.5 - 405}{9} = -37.5\ kN$



$\Sigma M_C = 0$

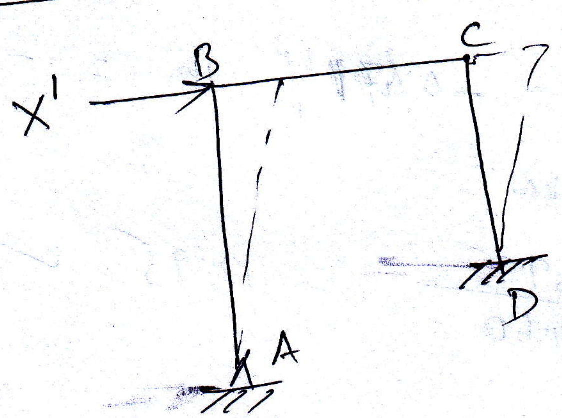
$\Rightarrow -H_D \times 6 + M_{DC} + M_{CD} = 0$

$H_D = \dfrac{M_{CD} + M_{DC}}{6} = 7.45\ kN$

$H_A + 10 \times 9 + H_D - X = 0$

$X = 59.95\ kN$

Sway moment distribution



$M^F_{AB} = 0$

$M^F_{BA} = -\dfrac{3EI\Delta}{L^2}$

$= -\dfrac{9EI\Delta}{9^2}$

$= -\dfrac{EI\Delta}{9}$

$M^F_{BC} = M^F_{CB} = 0$

$M^F_{CD} = M^F_{DC} = -\dfrac{6EI\Delta \times 2}{6^2} = -\dfrac{EI\Delta}{3}$

Let $EI\Delta = 54$

$M^F_{BA} = -6, \quad M^F_{CD} = M^F_{DC} = -18$

~~Non~~ Sway Moment distribution

Joint	A	B		C		D
Member	AD	BA	BC	CB	CD	DC
DF	1	0.43	0.57	0.5	0.5	0
FEM	0	−6	0	0	−18	−18
Bal	0	2.58	3.18	9	9	0
COF	0	0	3	1.59	0	3
Bal	0	−1.29	−1.71	−0.8	−0.8	0
COF	0	0	−0.4	−0.86	0	−0.4
Bal	0	+0.17	+0.23	+0.43	+0.43	0
Final end moment	0	−4.24	4.3	9.36	−9.37	−15.4

Then, $H_A = -0.57$ kN, $H_D = -4.13$ kN

$X' = 4.60$ kN

$MF = \dfrac{59.95}{4.60} = 12.95$

Hence

$M_{AB} = 0$

$M_{BA} = \cancel{8.\ldots}\ 12.59$

$M_{BC} = -11.82$

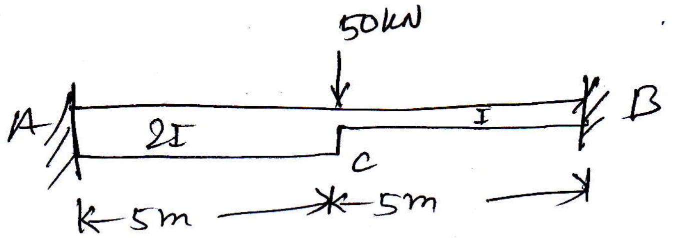
$M_{CB} = 91.29$

$M_{CD} = -91.42$

$M_{DC} = -184.65$

kNm

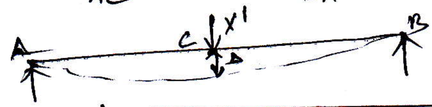
Q7



Non-sway moment distribution

$X = 50\ kN \uparrow$, take $EI\Delta = 25$, $X' \downarrow$

$M^F_{AC} = M^F_{CA} = -12$, $M^F_{CB} = M^F_{BC} = 6$



Joint	A	C		B
Member	AC	CA	CB	BC
RS	2I/5	2I/5	I/5	I/5
DF	0	0.67	0.33	0
FEM	-12	-12	6	6
Bal	0	4.0	2	0
COF	2	0	0	1
Bal	0	0	0	0
Final end moment	-10	-8	8	7

Also,

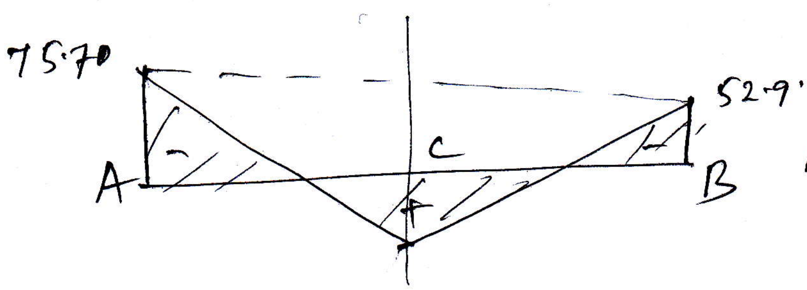
$R_A = 3.6\ kN$, similarly, $R_B = 3\ kN$

$X' = 6.6\ kN \downarrow$

$MF = \frac{X}{X'} = \frac{50}{6.6} = 7.57$

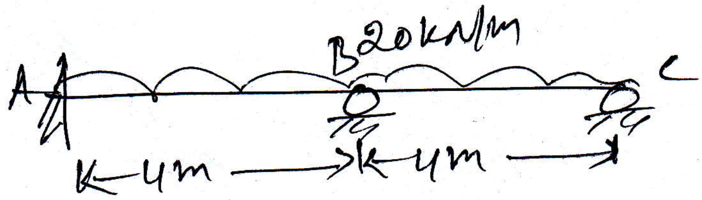
End moments are $M_{AC} = -75.7\ kNm$, $M_{CA} = -60.59\ kNm$

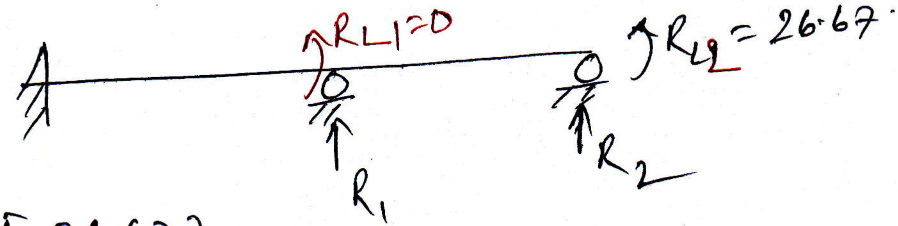
$M_{BC} = 52.9\ kNm$, $M_{CB} = 60.59\ kNm$



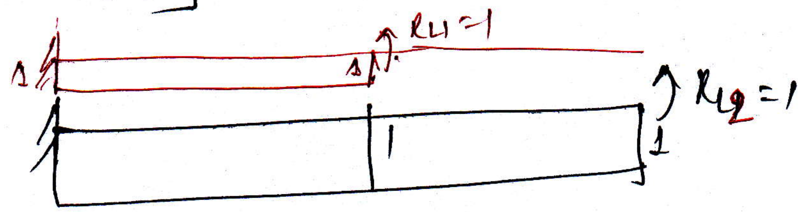
BMD

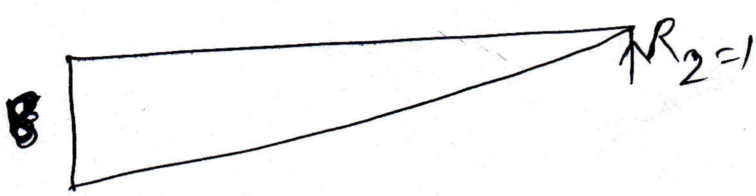
Q8. Use flexibility matrix method





$$q_M = \begin{bmatrix} -26.67 \\ +26.67 \\ -26.67 \\ +26.67 \end{bmatrix} \qquad R_L = \begin{bmatrix} 26.67 \end{bmatrix}$$





$$B = \begin{array}{cc|cc} \multicolumn{2}{c|}{B_L} & \multicolumn{2}{c}{B_R} \\ +1 & 1 & 4 & 8 \\ -1 & -1 & 0 & -4 \\ 0 & 1 & 0 & 4 \\ 0 & -1 & 0 & 0 \end{array}$$

$$f = \frac{4}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$B_R^T f = \frac{2}{3EI}\begin{bmatrix} 8 & -4 & 0 & 0 \\ 20 & -16 & 8 & -4 \end{bmatrix}$$

$$F_{RR} = B_R^T f \; B_R = \frac{2}{3EI}\begin{bmatrix} 32 & 80 \\ 80 & 256 \end{bmatrix}$$

$$F_{RR}^{-1} = \frac{3EI}{3584}\begin{bmatrix} 256 & -80 \\ -80 & 32 \end{bmatrix}$$

$$R = F_{RR}^{-1}\left[D - D_{RL}\right] R_L$$

$$D_{RL} = B_R^T f \; B_L = \frac{1}{EI}\begin{bmatrix} 213.3 \\ 853.55 \end{bmatrix} \cancel{\frac{1}{EI}\begin{bmatrix} 8 \\ 32 \end{bmatrix}}$$

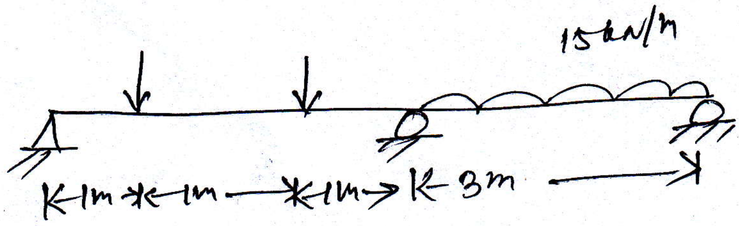
$$D = \frac{1}{EI}\begin{bmatrix} -300 \\ 0 \end{bmatrix}$$

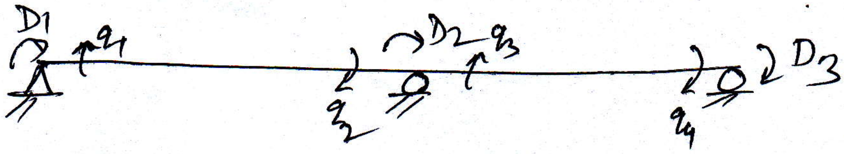
$$R = \begin{bmatrix} -52.80 \\ 11.52 \end{bmatrix}$$

$$q_0 = B_L R_L + B_R R = \cancel{\begin{bmatrix} -65.45 \\ -100.47 \\ +101.47 \\ -26.26 \end{bmatrix}} \begin{bmatrix} -92.62 \\ -72.74 \\ 72.74 \\ -26.67 \end{bmatrix}$$

$$q = q_0 + q_M = \begin{bmatrix} -119.28 \\ -46.08 \\ 46.08 \\ 0 \end{bmatrix} \cancel{\begin{bmatrix} -92.12 \\ -74.8 \\ +74.8 \\ 0 \end{bmatrix}} \text{ kNm}$$

Q9 stiffness matrix method





$$q_{f} = \begin{bmatrix} -33.33 \\ +33.33 \\ -11.25 \\ +11.25 \end{bmatrix} \text{kNm}$$

$$q_2 = \begin{bmatrix} -33.33 \\ 22.08 \\ 11.25 \end{bmatrix}$$

$$D = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}$$

Then $$[K] = \frac{EI}{3}\begin{bmatrix} 4 & 2 & 0 \\ 2 & 8 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

$$[K_M] = \frac{EI}{3}\begin{bmatrix} 4 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 4 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

$$K^{-1} = \frac{3}{EI}\begin{bmatrix} 7/24 & -1/12 & 1/24 \\ -1/12 & 1/6 & -1/12 \\ 1/24 & -1/12 & 7/24 \end{bmatrix}$$

$$[D] = K^{-1}\left[q_D - q_L\right]$$

$$= K^{-1}\left[\{0\} - \begin{bmatrix} -33.33 \\ 22.08 \\ 11.25 \end{bmatrix}\right]$$

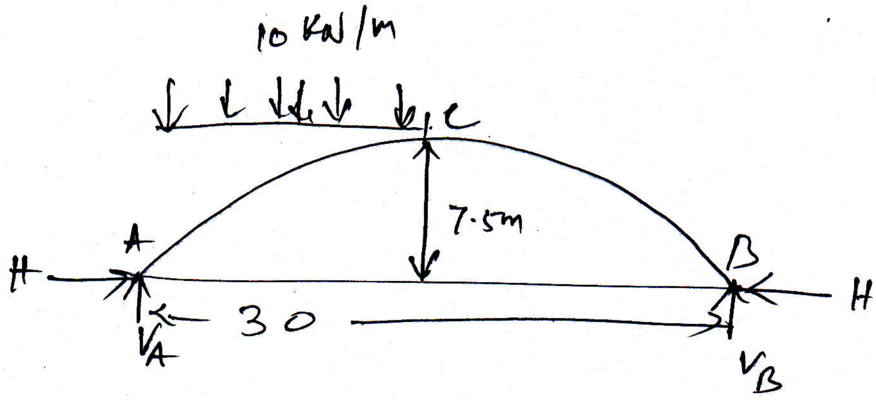
$$= \frac{-3}{EI}\begin{bmatrix} -11.09 \\ 5.52 \\ 0.0525 \end{bmatrix}$$

$$q_{KM} = K_M D = -\begin{bmatrix} 33.32 \\ -0.1 \\ 22.185 \\ -11.25 \end{bmatrix}$$

$$q = q_M + q_{KM}$$

$$q = \begin{bmatrix} 0 \\ 33.43 \\ -33.43 \\ 0 \end{bmatrix} \text{kNm}$$

Q10



$V_A = 112.5$ kN , $V_B = 37.5$ KN

$y = \frac{4h}{l^2}(lx - x^2) = \frac{1}{30}(30x - x^2)$

$$\int_0^{30} M_0 y \frac{dx}{EI_c} = \frac{1}{EI_c}\left[\int_0^{15}\left(V_A x - \frac{10x^2}{2}\right)\frac{1}{30}(30x - x^2)\,dx + \int_0^{15} V_B \cdot x \cdot \frac{1}{30}(30x - x^2)\,dx\right]$$

$$= 67500 / EI_c$$

$$\int_0^L y^2 \frac{dx}{EI_c} = \frac{8h^2L}{15EI_c} = \frac{900}{EI_c}$$

$H = 75$ kN

Position of Max^m BM

(i) Max^m +ve BM.

Take a section x from A.

$M_x = 112.5x - 5x^2 - \frac{75}{30}(30x - x^2)$

~~$= 112.5x - 5x^2 - 75$~~

$\frac{dM_x}{dx} = 0 \Rightarrow x = 7.5$ m

Max^m +ve BM $= 140.625$ kNm

when $x = 7.5$m from A, $y = \frac{1}{30}(30x - x^2)$

$\frac{dy}{dx} = \frac{1}{2}$, $\sin\theta = \frac{1}{\sqrt{5}}$, $\cos\theta = \frac{2}{\sqrt{5}}$

$N = 83.85 kN$, $F = 0$

(ii) Max^m -ve BM

Let x from B

$M_x = V_B x - Hy = 37.5x - \frac{75}{30}(30x - x^2)$

$\frac{dM_x}{dx} = 0 \Rightarrow 37.5 - \frac{75}{30}(30 - 2x) = 0$

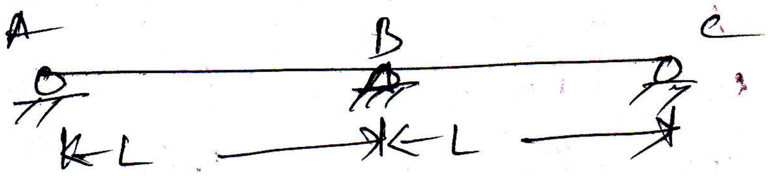
$30 - 2x = 15$

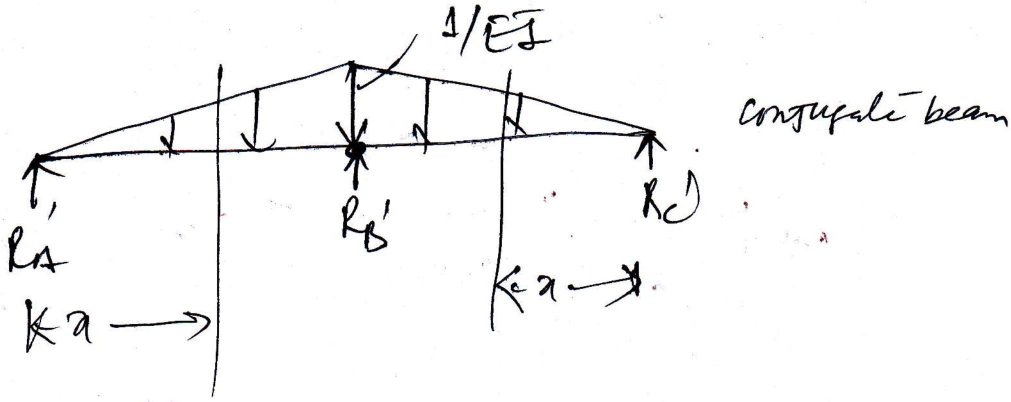
$x = 7.5 m$

~~Max~~ $(-BM)_{max} = -140.625$ kNm.

$N = 83.85 kN$, $F = 0$

Q11





$$R_A' = R_C' = \frac{L}{6EI}, \quad R_B' = \frac{2L}{3EI} = \phi_{BB}$$

$$M_x' = \frac{x}{6EI}\left(\frac{x^2 - L^2}{L}\right) = y'_{xB}$$

$$M_B = \frac{y'_{xB}}{\phi_{BB}} = \frac{x}{4l^2}\left(x^2 - L^2\right)$$

x from A	IL ordinates of M_B
0	0
0.25L	−0.0586L
0.5L	−0.09375 L
0.75L	−0.08203L
L	0
1.25 L	−0.08203L
1.5 L	−0.09375L
1.75 L	−0.0586L
2L	0